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The proposed research is to develop set-valued methods for robust nonlinear control, where "nonlinear control" refers to nonlinear controllers for both linear and nonlinear systems. Set-valued methods represent a natural framework for incorporating uncer-tainty into control system analysis and design. Uncertainty may come from unknown parameters, bounded disturbances, neglected dynamics, or uncertain state values. In a set-valued setting, system dynamics become set-valued, state estimates are set-valued, and feedback controls become set-valued. The proposed research is to explore thc util-ity of set-valued methods in the specific areas of (1) output feedback control of systems with saturations, (2) decentralized control, (3) nonlinear gain-scheduled control design, (4) adaptive control, and (5) control of hybrid dynamical systems. A primary objective throughout is the explicit computational construction of control laws which guarantee achievable optimal performance. The set-valued approach is sufficiently diverse to accommodate the formulation of a variety of control problems. The computational burden associated with set-valued methods can be significant, involving the solution of relatively smalll linear programs many times over. The spirit of the project has been to exploit computational power so that the design and implementation of set-valued methods can be a reality.

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Set-Valued Methods for Robust Nonlinear Control

Final Report for AFOSR F49620–97–1–0197

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1 Introduction

The following is the Project Summary from the original proposal:

> The proposed research is to develop set-valued methods for robust nonlinear control, where "nonlinear control" refers to nonlinear controllers for both linear and nonlinear systems. Set-valued methods represent a natural framework for incorporating uncertainty into control system analysis and design. Uncertainty may come from unknown parameters, bounded disturbances, neglected dynamics, or uncertain state values. In a set-valued setting, system dynamics become set-valued, state estimates are set-valued, and feedback controls become set-valued. The proposed research is to explore the utility of set-valued methods in the specific areas of (1) output feedback control of systems with saturations, (2) decentralized control, (3) nonlinear gain-scheduled control design, (4) adaptive control, and (5) control of hybrid dynamical systems. A primary objective throughout is the explicit computational construction of control laws which guarantee achievable optimal performance.

The set-valued approach is sufficiently diverse to accommodate the formulation of a variety of control problems. The computational burden associated with set-valued methods can be significant, involving the solution of relatively small linear programs many times over. The spirit of the project has been to *exploit* computational power so that the design and implementation of set-valued methods can be a reality.

2 Research Progress

The following describes the main research results over the funding period April 1997 through December 1998.

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2.1 Output Feedback for Systems with Constraints

An important issue in control systems is that of state and control constraints. Control constraints take the form of actuator saturations and rate limiters. State constraints may be imposed by modeling issues (e.g., remaining in a desired operating region over which a linear model is suitable) or by performance considerations (e.g., satisfying a tracking error bound).

There are a variety of control strategies which address systems with constraints, including heuristic mechanisms, such as integrator anti-windup, controller scheduling/switching, and receding horizon control. It is possible to provide sufficient conditions under which various strategies can meet imposed constraints.

In this work, we considered the following unsolved problem:

> *Given a set of state and control constraints, does there exist* any *output feedback controller which can satisfy the imposed constraints.*

This question addresses the problem of constraints at face value. Note that no underlying control structure is imposed *a priori*.

In the case of discrete-time scalar control problems, we were able to provide a computationally constructive solution to this problem. The system under consideration takes the form

$$x(k+1) = Ax(k) + B_1 d(k) + B_2 u(k)$$

$$z(k) = C_1 x(k) + D_{11} d(k) + D_{12} u(k)$$

$$y(k) = C_2 x(k) + n(k)$$

where u is the control input, y is the measured output, and d and n are process disturbances and measurement noises, respectively. The control objective (which can be used to impose state and control constraints) is to maintain $|z(k)| \le z_{\max}$ in the presence of disturbances and noises which, after normalization, satisfy $|d(k)| \le 1$ and $|n(k)| \le 1$.

By developing the PI's work in the state-feedback case and work in set-valued observers, we derived computational tests which determine whether any controller can meet prescribed constraints. Furthermore, we showed that the controller can be taken to be a set-valued observer (SVO) followed by a static selection function.

A SVO produces a *set* of state estimates, $x(k) \in X(k)$, which are consistent with a measured output trajectory. If the actual state were known, then existing state-feedback results can be used to produce a *set* of admissible control values, $u(k) \in R(x(k))$, which assure that the performance objective is achieved. Since the state is unknown, the control must satisfy

$$u(k) \in \bigcap_{\xi \in X(k)} R(\xi)$$

Computational tests were derived which assure that the above intersection is never empty, which in turn implies that under output feedback, the performance objective is achieved and hence the

constraints are not violated. If these tests fail, then no controller can achieve the desired performance.

2.2 Scheduling Nonlinear Controllers

In traditional gain-scheduling, a global nonlinear controller is pieced together using linear control designs based on fixed operating conditions. In some cases, it may be advantageous to used nonlinear controllers even for the fixed operating condition designs. For example, it may be that a linearized plant is not controllable. Alternatively, some control strategies produce nonlinear controllers for linear systems (in particular, set-valued methods).

This work considered gain-scheduling in which a global nonlinear controller is pieced together using local *nonlinear* controllers. A potential advantage of using nonlinear controllers is to alleviate two of the main limiting factors in traditional gain-scheduling, namely rapid transitions among operating conditions and neglected nonlinearities.

The PI's earlier work on set-valued methods for linear parameter varying (LPV) systems addressed systems of the form

$$x(k+1) = A(\theta(k))x(k) + B_1 d(k) + B_2(\theta(k))u(k)$$

$$z(k) = C_1(\theta(k))x(k) + D_{11}d(k) + D_{12}u(k)$$

$$|\theta(k+1) - \theta(k)| \leq \rho$$

The objective is to maintain $|z(k)| \leq z_{\max}$ in the presence of disturbance $|d(k)| \leq 1$. The variable θ is an exogenous parameter whose rate of variation is bounded by ρ. Using set-valued methods, it is possible to construct nonlinear parameter dependent state feedback $u(k) = g(x(k), \theta(k))$ to achieve the desired minimization.

LPV dynamics form the underlying basis of gain-scheduling with two major differences: 1) the "parameter" is actually an endogenous variable and 2) the state dynamics include high order nonlinearities. It is often possible to write the dynamics in the form

$$\theta(k+1) = \theta(k) + E(\theta(k))x(k)$$

$$x(k+1) = A(\theta(k))x(k) + B_1 d(k) + B_2(\theta(k))u(k) + Hv(k)$$

$$z(k) = C_1(\theta(k))x(k) + D_{11}d(k) + D_{12}u(k)$$

where $v(k)$ is an artificial disturbance which represents high-order nonlinearities such that

$$|v_i(k)| \leq \gamma_i |x(k)|^2$$

provided that

$$|Fx(k)| \leq 1$$

It is possible to use previous results on LPV systems as follows:

- Treat $\theta(k)$ as an "exogenous" parameter.
- Assume $|\theta(k+1) - \theta(k)| \leq \rho$ for some ρ.
- Impose $|E(\theta(k))| \leq \rho$ in the design.
- Include an artificial disturbance $v(k)$.
- Impose $|Fx(k)| \leq 1$ in the design.

In this manner, transition rates between operating conditions as well as high order nonlinearities are included in the design process—at the cost of increased computational requirements and possible conservatism. If the LPV control construction algorithm converges, then one obtains a nonlinear gain-scheduled controller $u(k) = g(\theta(k), x(k))$ which is guaranteed to achieve the desired performance. Unlike traditional gain-scheduling, stability and performance is built into the design process, thereby alleviating the need for extensive evaluative simulations.

Another research result addressed the stability of the nonlinear system

$$x(k+1) = f(x(k), d(k), \theta(k))$$

and showed that input-output stability for $\theta(k)$ "frozen" implies input-output stability for $\theta(k)$ slowly varying. Unlike classical results from differential equations, this work did not assume exponential stability in case $\theta(k)$ is frozen, thereby admitting the possibility of nonlinear controllers for systems whose linearizations are not controllable.

2.3 Adaptive Control with Guaranteed Transient Bounds

An important issue in adaptive control is the transient behavior during the "identification" phase in which a parameter estimator seeks to identify actual parameter values. This work considered the utility of set-valued methods in the preliminary state-feedback case.

The system dynamics take the form

$$x(k+1) = Ax(k) + B_1 d(k) + B_2 u(k) + E(x(k))\theta(k)$$

$$\theta(k+1) = \theta(k)$$

$$z(k) = C_1 x(k) + D_{11} d(k) + D_{12} u(k)$$

where $\theta(k)$ is a vector of constant unknown parameters. The objective is to maintain $|z(k)| \leq z_{\max}$ in the presence of disturbance $|d(k)| \leq 1$.

In this work, we combined methods for set-valued state feedback and SVO's to produce a sequence of parameter independent feedback laws $u(k) = g_i(x(k))$ which assure that $|z(k)| \leq z_{\max,i}$, where $z_{\max,i}$ is a sequence of progressively more stringent performance bounds.

Set-valued tools turn out to be well suited for this problem. The unknown parameter vector is assumed to lie initial in some set $\Theta(0)$. Set-valued tools for LPV systems can be used to design a

state feedback $u(k) = g_0(x(k))$ which assures $|z(k)| \leq z_{\max,0}$ for all *time-varying* $\theta \in \Theta(0)$. As the system evolves, a SVO can be used to construct progressively shrinking sets $\Theta(k)$ which contain the "true" parameter vector. Whenever $\Theta(k_1)$ reflects significant parameter identification. a new feedback law, $u(k) = g_1(x(k))$ is constructed which assures $|z(k)| \leq z_{\max,1}$ for all time-varying $\theta \in \Theta(k_1)$.

The procedure continues by designing a feedback law $u(k) = g_i(x(k))$ which assures $|z(k)| \leq z_{\max,i}$ for all time-varying $\theta \in \Theta(k_i)$. At all times, the state magnitudes are kept in check by using an "uncertainty equivalence" feedback which assures performance over an uncertain parameter set, as opposed to a certainty equivalence controller which simply employs the parameter estimate in a parameter-dependent feedback law.

2.4 Receding Horizon for Systems with Constraints

A primary motivation for receding horizon control is the existence of state and control constraints. Receding horizon control uses on-line optimization of a finite-horizon objective to construct a feedback law. For linear systems with quadratic or piecewise linear objective functions, state and control constraints can easily be included in the on-line optimizations. One major difficulty is whether satisfying the constraints over the specified finite-horizon implies that the constraints can be satisfied over the infinite-horizon future.

It is possible to combine set-valued methods with receding horizon control to alleviate this difficulty. The system under consideration is

$$x(k+1) = Ax(k) + Bu(k)$$

$$z(k) = Cx(k) + Du(k)$$

The receding horizon cost function is

$$\min_{u(k),\ldots,u(k+N)} \sum_{i=k}^{k+N} h(x(i), u(i))$$

subject to the constraint $|z(k)| \leq 1$. Here, $h(x, u)$ is a penalty function which, for computational purposes, can be quadratic or piecewise linear. The above optimization leads to an control sequence $\{u^*(k), \ldots, u^*(k+N)\}$. Let $\phi_{\mathrm{RH}}(x)$ denote the mapping from $x(k)$ to $u^*(k)$. Then the receding-horizon feedback law is $u(k) = \phi_{\mathrm{RH}}(x(k))$, which amounts to implementing only the first element of the optimal control sequence.

The problem with the above procedure is that it may not be possible to compute an optimal control sequence $\{u^*(k), \ldots, u^*(k+N)\}$ which satisfies the constraints $|z(k)| \leq 1$. By using set-valued methods, one can translate the desired infinite-horizon constraint $|z(k)| \leq 1$ to a pointwise-in-time constraint, $|\tilde{z}(k)| \leq 1$, where

$$\tilde{z}(k) = \tilde{C}x(k) + \tilde{D}u(k)$$

for appropriate $\tilde{C}$ and $\tilde{D}$. The advantage here is that $|\tilde{z}(k)| \leq 1$ implies 1) $|z(k)| \leq 1$ and 2) $|\tilde{z}(k+1)|$ is possible.

The above ideas were developed to produce a receding horizon computational scheme which guarantees infinite-horizon feasibility of constraints, has guaranteed stability properties, and approximates the optimal infinite horizon policy.

This work prompted a new direction of research regarding stochastic manufacturing lines.

2.5 Computational Capabilities and Simulation Studies

Set-valued methods are computationally intensive and likely to be applicable to low order systems only. However, with ever increasing computational power, the interpretation of low order changes. Furthermore, the standard engineering practice of breaking down high order problems into collections of low order problems is especially amenable to set-valued methods because of the guaranteed magnitude bounds set-valued methods provide.

The following simulation studies have been conducted over the course of this project:

- **Missile Autopilot** via LPV gain-scheduling with rapid changes in angle-of-attack and actuator constraints.
- **Compressor combustion control** via LPV gain-scheduling with compressor characteristic uncertainty.
- **Boiler-turbine control** via LTI set-valued methods with state constraints and actuator constraints.
- **VTOL aircraft control** via LPV gain-scheduling.
- **Manufacturing system scheduling** via receding-horizon control with state constraints and actuator constraints.

2.6 Publications

The following Doctoral theses from the Department of Aerospace Engineering acknowledge the support of AFOSR:

- Kuan-Yang Tu, *Set-valued Methods for Estimation*, May 1997.
- Kuan-Hsuang Tu, *Analysis and Design of Nonlinear Gain-Scheduled Control Systems*, December 1997.
- Pang Chen, *Applications of Gain-Scheduled Control in Power Systems and V/STOL Aircraft*, May 1998.
- Lance Carter, Linear Parameter Varying Representations for Nonlinear Control Design, December 1998.

- Chih-Hua Hsu, *Dynamic Scheduling of Manufacturing Systems*, December 1998.

The following publications/reports acknowledge the support of AFOSR:

- J.S. Shamma and K.-Y. Tu, "Output Feedback Control for Systems with Constraints and Saturations: Scalar Control Case", *Systems & Control Letters*, 1998.
- J.S. Shamma and K.-Y. Tu, "Set-Valued Methods for the Output Feedback Control of Systems with Control Saturations", *Proceedings of the 36th IEEE Conference on Decision and Control*, San Diego, CA, December1997.
- K.-H. Tu and J.S. Shamma, "Nonlinear gain-scheduled control design using set-valued methods", *Proceedings of the 1998 American Control Conference*, Philadelphia, PA, June 1998.
- K.-H. Tu and J.S. Shamma, "Nonlinear gain-scheduled control design using set-valued methods", submitted to *International Journal of Robust and Nonlinear Control.*
- C.-H. Hsu and J.S. Shamma, "Further results on linear non-quadratic optimal control", to appear in *IEEE Transactions on Automatic Control.*
- J.S. Shamma and D. Xiong, "Set-valuedmethods for linear parameter varying systems", *Automatica* 1999.
- J.S. Shamma, "Anti-windup via constrained regulation with observers", *Systems & Control Letters*, 1999.
- C.-H. Hsu and J.S. Shamma, "A Decomposition Approach to Scheduling of Failure Prone Transfer Lines," *System Theory: Modeling, Analysis, and Control*, T.E. Djaferis and I.C. Schick (eds), Kluwer Academic Publishers, 1999.
- J.S. Shamma, "Anti-windup via controlled invariance with observers", *Proceedings of the 1999 American Control Conference*, San Diego, CA, September 1998.